Thermodynamic Formalism and Phase Transitions of Generalized Mean–Field Quantum Lattice Models

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The general structure of thermodynamic equilibrium states for a class of quantum mechanical (multi–lattice) systems is elaborated, combining quantum statistical and thermodynamical methods. The quantum statistical formulation is performed in terms of recent operator algebraic concepts emphasizing the role of the permutation symmetry due to homogeneous coarse graining and employing the internal symmetries. The variational principle of the free energy functional is derived, which determines together with the symmetries the general form of the limiting Gibbs states in terms of their central decomposition. The limiting minimal free energy density and its possible equilibrium states are analyzed on various levels of the description by means of convex analysis, where the Fenchel transforms of the free energies provide entropy like potentials. On the thermodynamic level a modified entropy surface is obtained, which specifies only in combination with its concave envelope the regions of pure and mixed phase states. The symmetry properties of a certain model allow to specify the (non–) differentiability of the minimal free energy density. A characterization and classification of phase transitions in terms of quantum statistical equilibrium states is proposed, and the connection to the Landau theory is established demonstrating that the latter implies a (continuous) deformation of the sets of equilibrium states along a canonically given curve.

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